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DETERMINATION OF STRESS INTENSITY FACTORS AT THE TIPS OF CRACKS GROWING FROM LOADED HOLES IN FINITE ANISOTROPIC PLATES

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In pin, bolt, and rivet joints, stress concentration in combination with fretting between the fastening element and the surface of the hole may lead to the formation of damages and defects. In order to be able to predict the safe life of a structure, it is necessary to be able to precisely calculate the limit load and estimate the growth of defects near fastener holes in such joints. A survey of the studies done in this area for isotropic elastic plates can be found in [1, 2], for example. Progress is being made relatively slowly in regard to the investigation of the problem for plates made of composite materials (see the surveys in [3-5], for example). The reason for this is a shortage of information on the effect of the anisotropy of the material, the boundaries of the plate, and the type of load transmission on the stress intensity factors (SIF) at the tips of cracks near loaded holes.

In the present study, we construct special representations of the solution of problems involving determination of the elastic equilibrium of a finite rectilinear anisotropic plate with a system of through slits and a loaded elliptical hole. Automatic satisfaction of the boundary conditions at the contour of the hole makes it possible to reduce the problem to the solution of a system of integral equations (IE) whose order is one less than the number of components of the boundary of the region. The absence of an unknown function at the boundary of the hole makes it possible to more efficiently find numerical solutions. Using the example of a rectangular plate with cracks originating from the contour of a hole loaded through a pin, we study the effect of anisotropy of the material, a wide range of pin-joint geometries, and different combinations of load transmission from the pin and seat with interference on the value of the SIF at the tips of the cracks. Data for an isotropic material is obtained by taking the limit in the anisotropy parameters in a numerical solution.

We will examine an elastic, rectilinearly anisotropic plate of constant thickness h bounded by closed contours Λ (an ellipse with the semiaxes a and b) and L_0 (smooth longitudinal external contour) and having n smooth internal through slits (cracks) L_j ($j = \overline{1, n}$). The plate is loaded by a self-balanced system of external forces applied to L_0 and Λ. The edges of the slits $L' = \bigcup_{j=1}^{n} L_j$ are not loaded. We will make the axes of symmetry of the ellipse coincide with the axes of the Cartesian coordinate system xOy. As the positive direction on L_0 we take the direction which leaves the plate on the left. On the slit L_j, with ends a_j and b_j, the positive direction leads from a_j to b_j. We direct the normal $\mathbf{n}$ to the

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right with positive circumvention $L = \bigcup_{j=0}^{n} L_j$. A generalization to the case of edge notches is given below.

The stress state of the plate can be expressed through two functions $\Phi_\nu(z_\nu)$ ($z_\nu = x + \mu_\nu y$, $\nu = 1, 2$) [6]

$$(\sigma_x, \tau_{xy}, \sigma_y) = 2\mathrm{Re}\left\{\sum_{\nu=1}^{2} \left(\mu_\nu^2, -\mu_\nu, 1\right) \Phi_\nu(z_\nu)\right\}, \tag{1}$$

satisfying the boundary conditions [7, 8]:

$$\begin{gathered} a(t)\Phi^+{}_1(t_1) + b(t)\overline{\Phi^+{}_1(t_1)} + \Phi^+{}_2(t_2) = F(t), \\ F(t) = [X_n(t) + \bar{\mu}_2 Y_n(t)][(\mu_2 - \bar{\mu}_2)M_2(t)]^{-1},\ t \in \Lambda \cup L_0; \end{gathered} \tag{2}$$

$$a(t)\Phi_1^\pm(t_1) + b(t)\overline{\Phi_1^\pm(t_1)} + \Phi_2^\pm(t_2) = 0,\ t \in L'. \tag{3}$$

Here and below, we use the notation from [8]; the superscript + (−) denotes the limiting value of functions as they approach the contour from the left (right); $X_n(t)$, $Y_n(t)$ are projections of the external forces at the point $t \in \Lambda \cup L_0$ on the x, y axes; μ_ν are roots of the characteristic equation.

Using the results in [8], we represent the sought functions as

$$\begin{gathered} \Phi_\nu(z_\nu) = \sum_{j=0}^{1} \Phi_\nu^j(z_\nu), \\ \Phi_\nu^1(z_\nu) = \left[2\pi i \omega_\nu'(\zeta_\nu)\right]^{-1} \int_L \left\{\frac{\Omega_\nu(\tau)\, d\tau_\nu}{\eta_\nu - \zeta_\nu} + \frac{l_\nu \overline{\Omega_1(\tau)}\, d\bar{\tau}_1}{\zeta_\nu(\zeta_\nu \bar{\eta}_1 - 1)} + \frac{n_\nu \overline{\Omega_2(\tau)}\, d\bar{\tau}_2}{\zeta_\nu(\zeta_\nu \bar{\eta}_2 - 1)}\right\}, \end{gathered} \tag{4}$$

where $\Omega_\nu(t) = \{\Omega_{\nu j}(t) | t \in L_j,\ j = \overline{0, n}\}$ are unknown complex functions; $\Phi_\nu{}^2(z_\nu)$ is the solution for an infinite anisotropic plate with an elliptical hole subjected to specified external forces about its contour.

We will use $r(\gamma)$ and $q(\gamma)$ to represent the projections of the external forces on a normal and a tangent to the contour $\Lambda = \{z = a\cos\gamma - ib\sin\gamma | 0 \le \gamma < 2\pi\}$. To avoid the computational problems that would arise if $\Phi_\nu{}^0(z_\nu)$ were represented in the form of an infinite series or integrals of the Cauchy type [6], we will approximate $r(\gamma)$ and $q(\gamma)$ by piecewise-constant expressions

$$\begin{gathered} r(\gamma) \approx r_j,\ q(\gamma) \approx q_j,\ \gamma_{j-1} \leqslant \gamma \leqslant \gamma_j,\ \gamma_j = j\Delta\gamma, \\ \Delta\gamma = 2\pi/k,\ j = \overline{1, k}. \end{gathered}$$

This can always be done with a prescribed degree of accuracy given sufficiently large k. Then after performing certain transformations we obtain the following closed analytical representations for $\Phi_\nu{}^0(z_\nu)$:

$$\begin{gathered} \Phi_\nu^0(z_\nu) = \left[z_\nu'(\xi_\nu)\right]^{-1} \sum_{j=1}^{k} \{r_j \Phi_{\nu 1}^j(z_\nu) + q_j \Phi_{\nu 2}^j(z_\nu)\}, \\ \Phi_{\nu p}^j(z_\nu) = -\frac{A_{\nu j}^p}{\xi_\nu} + \frac{M_{\nu j}^p}{(\mu_\nu - \mu_{3-\nu})(\sigma_j - \xi_\nu)} - \frac{i}{4\pi(\mu_\nu - \mu_{3-\nu})}\Big\{[a_p\Delta_j + b_p\delta_j]\, s_{\nu j} + \\ + b_{\nu p}[\ln t_{\nu j} - \xi_\nu s_{\nu j}] + c_{\nu p}\left[\frac{s_{\nu j}}{\xi_\nu} + \frac{1}{\xi_\nu^2}(\ln t_{\nu j} - \ln m_j)\right]\Big\}, \\ a_1 = -a\mu_{3-\nu},\ a_2 = a,\ b_1 = ib,\ b_2 = ib\mu_{3-\nu},\ \mu_\nu = \alpha_\nu + i\beta_\nu, \\ \Delta_j = \sigma_{j-1} + \frac{1}{\sigma_{j-1}},\ \delta_j = \sigma_{j-1} - \frac{1}{\sigma_{j-1}},\ s_{\nu j} = \frac{\sigma_j - \sigma_{j-1}}{(\sigma_j - \xi_\nu)(\sigma_{j-1} - \xi_\nu)}, \\ b_{\nu 1} = ib - a\mu_{3-\nu},\ b_{\nu 2} = a + ib\mu_{3-\nu},\ t_{\nu j} = \frac{\sigma_{j-1} - \xi_\nu}{\sigma_j - \xi_\nu},\ c_{\nu 1} = ib + a\mu_{3-\nu}, \\ c_{\nu 2} = ib\mu_{3-\nu} - a,\ m_j = \sigma_{j-1}/\sigma_j, \end{gathered} \tag{5}$$

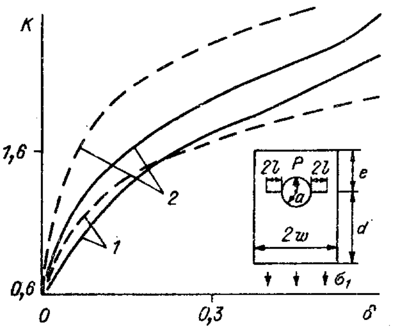

Fig. 1

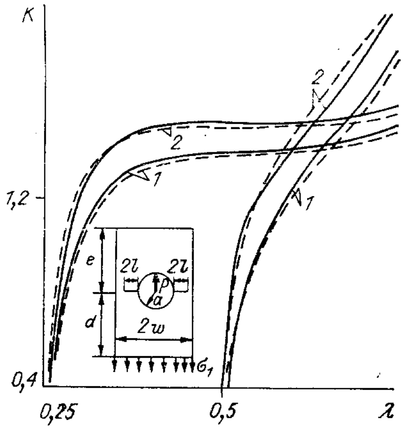

Fig. 2

$$M^{p}_{\nu j}=2i\left[\mu_{3-\nu}\,\mathrm{Im}\,(A^{p}_{1j}+A^{p}_{2j})-\sum_{\mu=1}^{2}(\beta_\mu \mathrm{Re}\,A^{p}_{\nu j}+\alpha_\mu\,\mathrm{Im}\,A^{p}_{\mu j})\right], \tag{5}$$

$$z_\nu=z_\nu(\xi_\nu)=\frac{a+i\mu_\nu b}{2}\xi_\nu+\frac{a-i\mu_\nu b}{2}\frac{1}{\xi_\nu},\quad |\xi_\nu|<1,$$

$$\xi_\nu=\xi_\nu(z_\nu)=\frac{z_\nu-\sqrt{z_\nu^2-(a^2+\mu_\nu^2 b^2)}}{a+i\mu_\nu b},\quad \xi_\nu(\infty)=0,$$

where $\sigma_j = \exp(-i\gamma_j)$, while $A_{\nu j}{}^{p}$ are determined from the system of equations

$$\sum_{\nu=1}^{2}\left(\mu_\nu^{k-2}A^{p}_{\nu j}-\overline{\mu_\nu^{k-2}A^{p}_{\nu j}}\right)=b^{p}_{k}/(2\pi i)\quad (k=\overline{1,4}),$$

$$b^{p}_{1}=\frac{-a_{12}b^{p}_{3}+a_{26}b^{p}_{2}}{a_{22}},\quad b^{1}_{2}=\frac{a}{2}(\Delta_{j+1}-\Delta_j),\quad b^{2}_{2}=\frac{ib}{2}(\delta_j-\delta_{j+1}),$$

$$b^{1}_{3}=-b^{2}_{2},\quad b^{2}_{3}=-b^{1}_{2},\quad b^{p}_{4}=\frac{a_{16}b^{p}_{3}-a_{12}b^{p}_{2}}{a_{11}}.$$

Here, $\Phi_{\nu p}{}^{j}(z_\nu)$ are solutions for a plate with a hole loaded on the part $[z(\gamma_{j-1}), z(\gamma_j)]$ of the contour Λ by unit normal forces ($p = 1$) and shearing forces ($p = 2$).

The thus-constructed potentials $\Phi_\nu(z_\nu)$ (4), (5) automatically satisfy boundary conditions (2) on Λ. Following [7], we take

$$\Omega_2(t)=-a(t)\Omega_1(t)+b(t)\overline{\Omega_1(t)},\ t\in L_0. \tag{6}$$

Inserting limiting values of $\Phi_\nu(z_\nu)$ from (4) into boundary conditions (2)-(3), considering (6), and performing certain transformations, we obtain a system of singular IE on L' and Fredholm IE of the second kind on L_0 for the sought functions $\Omega_1(t)$:

$$2\delta b(t)+\int_L\{K_1(t,\tau)\Omega_1(\tau)+K_2(t,\tau)\overline{\Omega_1(\tau)}\}\,ds=f(t),$$

$$K_1(t,\tau)\,ds=\frac{a(t)}{\pi i\omega_1'(\zeta_1)}\left[\frac{d\tau_1}{\eta_1-\zeta_1}+\frac{(-1)^{\delta}n_1\overline{b(\tau)}\,d\bar\tau_2}{\zeta_1(1-\zeta_1\bar\eta_2)}\right]+\frac{b(t)}{\pi i\overline{\omega_1'(\zeta_1)}}\left[\frac{\bar l_1 d\bar\tau_1}{\bar\zeta_1(1-\bar\zeta_1\eta_1)}-\right.$$

$$\left.-\frac{n_1 a(\tau)\,d\tau_2}{\bar\zeta_1(1-\bar\zeta_1\eta_2)}\right]-\frac{1}{\pi i\omega_2'(\zeta_2)}\left[\frac{a(\tau)\,d\tau_2}{\eta_2-\zeta_2}-\frac{(-1)^{\delta}n_2\overline{b(\tau)}\,d\bar\tau_2}{\zeta_2(1-\zeta_2\bar\eta_2)}\right], \tag{7}$$

$$K_2(t,\tau)\,ds=\frac{a(t)}{\pi i\omega_1'(\zeta_1)}\left[\frac{l_1 d\bar\tau_1}{\zeta_1(\zeta_1\bar\eta_1-1)}+\frac{n_1\overline{a(\tau)}\,d\bar\tau_2}{\zeta_1(1-\zeta_1\bar\eta_2)}\right]-\frac{b(t)}{\pi i\overline{\omega_1'(\zeta_1)}}\left[\frac{d\bar\tau_1}{\bar\eta_1-\bar\zeta_1}+\right.$$

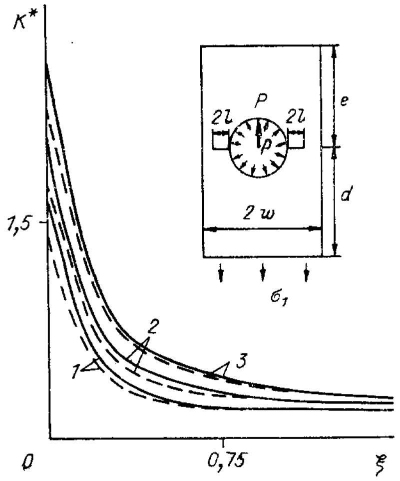

Fig. 3

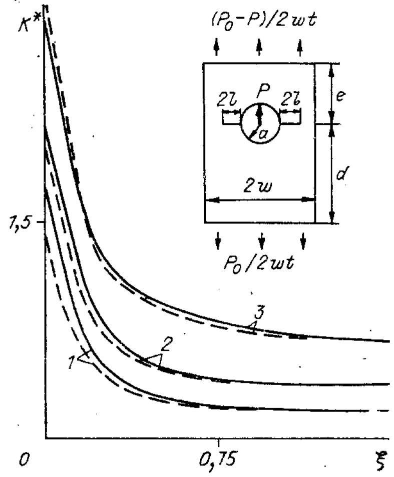

Fig. 4

$$+\frac{(-1)^{\delta}\bar{n}_1 b(\tau)\,d\tau_2}{\bar{\zeta}_1(1-\bar{\zeta}_1\eta_2)}\Bigg]-\frac{1}{\pi i\omega_2'(\zeta_2)}\left[\frac{(-1)^{\delta}b(\tau)\,d\tau_2}{\eta_2-\zeta_2}+\frac{l_2 d\bar{\tau}_1}{\zeta_2(1-\zeta_2\bar{\eta}_1)}-\frac{n_2\overline{a(\tau)}\,d\bar{\tau}_2}{\zeta_2(1-\zeta_2\bar{\eta}_2)}\right], \tag{7}$$

$$f(t)=2\{\delta F(t)-a(t)\Phi_1^0(t_1)-b(t)\overline{\Phi_1^0(t_1)}-\Phi_2^0(t_2)\},$$

where ds is an element of length of the arc L; $\delta = 0(1)$ at $t \in L'(L_0)$.

We need to add to Eqs. (7) conditions for the nonambiguity of the displacements in the circumvention of each slit

$$\int_{L_j}\Omega_{1j}(\tau)\,d\tau_1=0. \tag{8}$$

Following [7], we can show that the solution of Eq. (7) with additional conditions (8) in the class of functions

$$\Omega_j(t)=\Omega^j(t)[(t-a_j)(t-b_j)]^{-1/2},\ j=\overline{1,\ n},$$
$$\Omega^j(t)\in H(L_j),\ \Omega_0(t)\in H(L_0)$$

[$H(L_j)$ are bounded functions which are continuous on L_j in accordance with Holder's condition] exists and is unique. Using the Gauss–Chebyshev formulas for the integrals over L' and the rectangle formulas for the integrals over L_0, we reduce the solution of IE (7), (8) to the solution of a system of linear algebraic equations relative to the approximate values of the sought functions $\Omega^j(t)$, $\Omega_0(t)$ at the nodal points. Having solved it, we can use potentials (1), (4), and (5) and the formulas in [8] to find the stress distribution in the plate and the SIF for normal rupture and shear $K_1 = \lim_{t\to c} \sigma_n\sqrt{2\pi r}$, $K_2 = \lim_{t\to c} \tau_n\sqrt{2\pi r}$ at the tip c of a crack ($r = |t - c|$, t is a point lying on the extension of the crack past the end c on a tangent). In the case where a crack is present on the contour of an internal hole, the potentials (4), Eqs. (7)-(8), and the algorithm for the numerical solution of the IE should be modified appropriately [8].

Presented below are certain applications of the above solutions in regard to evaluation of the SIF at the tips of edge cracks growing from the contour of a free or loaded hole of radius a with its center at the origin of the coordinates in a rectangular plate ($L_0 = \{x = \pm w;\ -d < y < e\}$).

Let two edge cracks $L_{1,2} = \{t = \tau^{1,2}(\beta) = \pm[a + \ell(1 + \beta)]\,|\,|\beta| < 1\}$ originate from the contour of a circular hole. The plate is loaded through a rigid pin inserted without an allowance into the hole with the force $P = 2ah\sigma$ (σ is the crushing strength) along the y axis. Self-balanced, uniformly distributed forces $\sigma_1 = P_1/(2wh)$ are applied to the lower edge of the plate $y = -d$. To simplify the problem, we assume that friction is absent in the

TABLE 1

Case No.	η	δ 0,08	0,09	0,10	0,14	0,20	0,21	0,28
1	1,0	1,666/1,633	1,736/1,711	1,799/1,782	2,010/2,008	2,251/—	2,285/2,293	2,511/2,523
	2,0	1,153/1,119	1,200/1,171	1,241/1,216	1,375/—	1,516/1,501	1,534/—	1,657/—
2	1,0	0,798/0,790	0,852/0,836	0,879/0,879	1,031/1,027	1,211/—	1,239/1,240	1,428/1,432
	2,0	0,637/0,611	0,665/0,647	0,701/0,678	0,802/—	0,933/0,926	0,953/—	1,082/—
3	1,0	1,882/1,879	1,963/1,963	2,057/2,040	2,279/2,279	2,525/—	2,559/2,574	2,781/2,799
	2,0	1,205/1,197	1,249/1,248	1,289/1,291	1,425/—	1,553/1,304	1,570/—	1,677/—

Case No.	η	δ 0,30	0,35	0,40	0,42	0,49	0,50	0,56	0,60	0,63
1	1,0	2,571/—	2,715/2,731	2,855/—	2,909/2,932	3,099/3,131	3,126/—	3,285/3,330	3,391/—	3,461/3,531
	2,0	1,689/1,673	1,767/—	1,842/1,825	1,873/—	1,991/—	2,009/1,990	2,127/—	2,217/2,179	2,293/—
2	1,0	1,481/—	1,612/1,620	1,746/—	1,800/1,812	1,996/2,015	2,025/—	2,205/2,232	2,333/—	2,433/2,471
	2,0	1,117/1,112	1,205/—	1,295/1,290	1,332/—	1,470/—	1,482/1,486	1,630/—	1,735/1,716	1,823/—
3	1,0	2,838/—	2,975/3,002	3,107/—	3,158/3,196	3,331/3,382	3,356/—	3,499/3,568	3,593/—	3,662/3,754
	2,0	1,704/1,711	1,771/—	1,839/1,846	1,868/—	1,976/—	1,993/1,999	2,104/—	2,189/2,180	2,262/—

Note. A bar for the denominator denotes a lack of data.

TABLE 2

η	δ	M: 10	20	30	40
1,0	0,1	2,008	2,023	2,034	2,057
	0,6	3,313	3,356	3,572	3,593
2,0	0,1	1,290	1,299	1,299	1,299
	0,6	1,889	2,159	2,188	2,189

region of contact of the pin and the plate. The transmission of force from the pin to the plate will be modeled by a) a normal pressure σ distributed over a half-circle $\pi < \gamma < 2\pi$; b) a pressure distributed in accordance with the sine law $\sigma(t) = (4\sigma/\pi)|\sin\gamma|(\pi < \gamma < 2\pi)$.

Then, unless otherwise specified, the elastic parameters of the plate material E_1 = 53.84 GPa, E_2 = 17.95 GPa, G_{12} = 8.63 GPa, and ν_1 = 0.25. The solid (dashed) lines in the figures pertain to the case when the angle φ formed by the principal direction of anisotropy E_1 and the x axis is equal to 0 ($\pi/2$).

For case (b), Fig. 1 shows the results of calculations of corrected SIF values for normal rupture $K = K_1/K^*$ ($K^* = \sigma\sqrt{\pi(a + 2\ell)}$ — the SIF for normal rupture at the tips of an "equivalent" crack of the length $L^* = 2(a + 2\ell)$ in an infinite plate subjected to tension by forces $\sigma_y^\infty = \sigma$) in relation to $\delta = 2\ell/a$ with w/a = 2; d/a = 4; 6 and e/a = 4; 2 (curves 1 and 2, respectively). A decrease in the distance from the center of the hole to the unloaded edge of the plate markedly increases the SIF (≈25-50%). The increase is especially substantial if the principal direction of anisotropy corresponding to E_1 ($E_1 > E_2$) coincides with the y axis ($\varphi = \pi/2$).

Figure 2 shows the dependence of K on $\lambda = (a + 2\ell)/w$ for cases (a) and (b) (curves 1 and 2, respectively) with e = d = 2w, w/a = 4; 2. The angle φ has a slight effect on the value of K in the investigated ranges of the parameters. Uniform pressure (case a, curves 1) causes the SIF for normal rupture K_1 to be approximately 10-30% less than the values of the SIF for the case of distribution in accordance with a sine law (case b, curves 2). Such a reduction can evidently be attributed to the effect of local pressure along the axis of the crack. The calculations showed that the effect of this distribution law is particularly great for short cracks. Thus, the assumptions made in regard to the type of force distribution on the pin may result in a substantial difference between theoretical estimates of residual life and actual experimental data.

In many design schemes, loads arise on the surface of the hole due to pressure from the pin distributed over the surface $\sigma(t)$ in combination with a constant pressure p over the contour of the hole. This situation is seen, for example, in the case of a fastener installed with a negative allowance. For the case (b), Fig. 3 shows the dependence of the correction factor $K^* = K_1(\sigma_1\sqrt{2\pi\ell})^{-1}$ on $\xi = \ell/a$ with w/a = 10; e/w = d/w = 1 at p = 0; 0.4 σ; σ (curves 1-3). An increase in the negative allowance p is accompanied by a substantial increase in K^*. This is particularly evident for short cracks ($\delta < 0.5$).

In joints with a large number of pins (in structures composed of sheets joined together with a large number of rivets or bolts), only part of the entire load taken up by the joint can be transmitted through a given pin (bolt, rivet). Figure 4 shows the effect of the proportion of the force transmitted through the pin. The dependence of K^* on ξ was constructed for different ratios of the transmitted load P/P_0 = 1; 0.4; 0.2 (curves 1-3) with w/a = 10; e/w = d/w = 1 and an orthotropic plate material.

To evaluate the convergence of the algorithm and check the reliability of the results, we will examine three cases of loading of an isotropic rectangular plate (e = d = H): 1) uniformly distributed forces σ are applied to the lower and upper edges of the plate y = ±H (uniaxial tension); 2) a uniform normal pressure is applied to the contour of the hole; 3) a normal pressure $\sigma(t) = (4\sigma/\pi)|\sin\gamma|$ $(0 < \gamma < 2\pi)$ distributed according to a sine law is applied to the contour of the hole. Data for an isotropic material was found by taking the limit in the anisotropy parameters in a numerical solution (in the calculations, we assumed that $\mu_1 = 0.998i$; $\mu_2 = 1.002i$).

For comparison, Table 1 shows normalized values of the SIF K obtained on a BÉSM-6 computer within the framework of the proposed method (in the numerator) using L_0 M = 20 nodes on one-fourth of the boundary and the corresponding results from [9] (in the denominator) obtained by the collocation method for $w/a = 2$ and different $\delta = 2\ell/a$ and $\eta = H/w$. In all cases, the number of Chebyshev nodes M_1 on the contour of the cracks was assumed to be equal to 10. Fully satisfactory agreement is seen.

For case 3, Table 2 shows values of K^* at M = 10; 20; 30; 40; $M_1 = 10$; $\eta = 1$; 2 and $\delta = 0.1$; 0.6. The calculations show that even the use of the simplest method of discretizing the boundary L_0 (uniform subdivision) and rectangle integration formulas makes it possible to obtain a stable count and good convergence in the approximate solution for both short and long cracks. The values of SIF in Figs. 1-4 coincide to within the first two significant figures even with the number of nodes on half of the boundary L_0 $N \geq 50$ and on the contour of the crack $M_1 \geq 10$.

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